SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)

2013

Syllabus for PEA (Mathematics), 2013

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample Questions for PEA (Mathematics), 2013

1. Let $f(x) = \frac{1-x}{1+x}, x \neq -1$. Then $f(f(\frac{1}{x})), x \neq 0$ and $x \neq -1$, is
   (A) 1,
   (B) x,
   (C) x^2,
   (D) $\frac{1}{x}$.

2. The limiting value of $\frac{1.2+2.3+\ldots+n(n+1)}{n^3}$ as $n \to \infty$ is,
   (A) 0,
   (B) 1,
   (C) 1/3,
   (D) 1/2.

3. Suppose $a_1, a_2, \ldots, a_n$ are n positive real numbers with $a_1 a_2 \ldots a_n = 1$. Then the minimum value of $(1 + a_1)(1 + a_2)\ldots(1 + a_n)$ is
   (A) 2^n,
   (B) 2^{2n},
   (C) 1,
   (D)None of the above.

4. Let the random variable X follow a Binomial distribution with parameters n and p where $n(> 1)$ is an integer and $0 < p < 1$. Suppose further that the probability of $X = 0$ is the same as the probability of $X = 1$. Then the value of p is
   (A) $\frac{1}{n}$,
   (B) $\frac{1}{n+1}$,
   (C) $\frac{n}{n+1}$,
   (D) $\frac{n-1}{n+1}$.

5. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100})$ is
   (A) 1,
   (B) 2^{100},

(C) 0,

(D) None of the above.

6. If α and β are the roots of the equation $x^2 - ax + b = 0$, then the quadratic equation whose roots are $\alpha + \beta + \alpha\beta$ and $\alpha\beta - \alpha - \beta$ is

(A) $x^2 - 2ax + a^2 - b^2 = 0$,

(B) $x^2 - 2ax - a^2 + b^2 = 0$,

(C) $x^2 - 2bx - a^2 + b^2 = 0$,

(D) $x^2 - 2bx + a^2 - b^2 = 0$.

7. Suppose $f(x) = 2(x^2 + \frac{1}{x^2}) - 3(x + \frac{1}{x}) - 1$ where x is real and $x \neq 0$. Then the solutions of $f(x) = 0$ are such that their product is

(A) 1,

(B) 2,

(C) -1,

(D) -2.

8. Toss a fair coin 43 times. What is the number of cases where number of 'Head'$>$ number of 'Tail'?

(A) 2^{43},

(B) $2^{43} - 43$,

(C) 2^{42},

(D) None of the above.

9. The minimum number of real roots of $f(x) = |x|^3 + a|x|^2 + b|x| + c$, where a, b and c are real, is

(A) 0,

(B) 2,

(C) 3,

(D) 6.

10. Suppose $f(x, y)$ where x and y are real, is a differentiable function satisfying the following properties:

(i) $f(x+k, y) = f(x, y) + ky$;

(ii) $f(x, y+k) = f(x, y) + kx$; and

(iii) $f(x, 0) = m$, where m is a constant.

Then $f(x, y)$ is given by

(A) $m + xy$,

(B) $m + x + y$,

(C) mxy,

(D) None of the above.

11. Let $I = \int_{2}^{343} \{x - [x]\}^2 dx$ where $[x]$ denotes the largest integer less than or equal to x. Then the value of I is

(A) $\frac{343}{3}$,

(B) $\frac{343}{2}$,

(C) $\frac{341}{3}$,

(D) None of the above.

12. The coefficients of three consecutive terms in the expression of $(1+x)^n$ are 165, 330 and 462. Then the value of n is

(A) 10,

(B) 11,

(C) 12,

(D) 13.

13. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in

(A) $[\frac{1}{2}, 1]$,

(B) $[-1, 1]$,

(C) $[-\frac{1}{2}, \frac{1}{2}]$,

(D) $[-\frac{1}{2}, 1]$.

14. Let the function $f(x)$ be defined as $f(x) = |x-4| + |x-5|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at all points,

(B) $f(x)$ is differentiable at $x=4$, but not at $x=5$,

(C) $f(x)$ is differentiable at $x=5$ but not at $x=4$,

(D) None of the above.

15. The value of the integral $\int_0^1 \int_0^x x^2 e^{xy} dx dy$ is

(A) e,

(B) $\frac{e}{2}$,

(C) $\frac{1}{2}(e-1)$,

(D) $\frac{1}{2}(e-2)$.

16. Let $\mathcal{N}=\{1,2,\ldots\}$ be a set of natural numbers. For each $x \in \mathcal{N}$, define $A_n=\{(n+1)k, k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_2,

(B) A_4,

(C) A_5,

(D) A_6.

17. $\lim\limits_{x \to 0}\{\frac{1}{x}(\sqrt{1+x+x^2}-1)\}$ is

(A) 0,

(B) 1,

(C) $\frac{1}{2}$,

(D) Non-existent.

18. The value of $\binom{n}{0}+2\binom{n}{1}+3\binom{n}{2}+\ldots+(n+1)\binom{n}{n}$ equals

(A) 2^n+n2^{n-1},

(B) 2^n-n2^{n-1},

(C) 2^n,

(D) 2^{n+2}.

19. The rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is

(A) 1,
(B) 2,
(C) 3,
(D) 4.

20. Suppose an odd positive integer $2n + 1$ is written as a sum of two integers so that their product is maximum. Then the integers are

(A) $2n$ and 1,
(B) $n + 2$ and $n - 1$,
(c) $2n - 1$ and 2,
(D) None of the above.

21. If $|a| < 1, |b| < 1$, then the series $a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\ldots\ldots$ converges to
(A) $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$,
(B) $\frac{a(a+b)}{1-a(a+b)}$,
(C) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$,
(D) $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

22. Suppose $f(x) = x^3 - 6x^2 + 24x$. Then which of the following statements is true?
(A) $f(x)$ has a maxima but no minima,
(B) $f(x)$ has a minima but no maxima,
(C) $f(x)$ has a maxima and a minima,
(D) $f(x)$ has neither a maxima nor a minima.

23. An urn contains 5 red balls, 4 black balls and 2 white balls. A player draws 2 balls one after another with replacement. Then the probability of getting at least one red ball or at least one white ball is

(A) $\frac{105}{121}$,

(B) $\frac{67}{121}$,

(C) $\frac{20}{121}$,

(D) None of the above.

24. If $log_t x = \frac{1}{t-1}$ and $log_t y = \frac{t}{t-1}$, where $log_t x$ stands for logarithm of x to the base t. Then the relation between x and y is

(A) $y^x = x^{1/y}$,
(B) $x^{1/y} = y^{1/x}$,
(C) $x^y = y^x$,
(D) $x^y = y^{1/x}$.

25. Suppose $\frac{f''(x)}{f'(x)} = 1$ for all x. Also, $f(0) = e^2$ and $f(1) = e^3$. Then $\int_{-2}^{2} f(x)dx$ equals
(A) $2e^2$,
(B) $e^2 - e^{-2}$,
(C) $e^4 - 1$,
(D) None of the above.

26. The minimum value of the objective function $z = 5x + 7y$, where $x \geq 0$ and $y \geq 0$, subject to the constraints

$2x + 3y \geq 6, \;\; 3x - y \leq 15, \;\; -x + y \leq 4$, and $2x + 5y \leq 27$
is

(A) 14,
(B) 15,
(C) 25,
(D) 28.

27. Suppose A is a 2×2 matrix given as $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$.
Then the matrix $A^2 - 3A - 13I$, where I is the 2×2 identity matrix, equals

(A) I,
(B) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$,
(C) $\begin{pmatrix} 1 & 5 \\ 3 & 0 \end{pmatrix}$,

(D) None of the above.

28. The number of permutations of the letters $a, \; b, \; c,$ and d such that b does not follow a, c does not follow b, and d does not follow c is
(A) 14,
(B) 13,
(C) 12,
(D) 11.

29. Given n observations $x_1, \; x_2, \; \ldots, x_n$, which of the following statements is true ?
(A) The mean deviation about arithmetic mean can exceed the standard deviation,
(B) The mean deviation about arithmetic mean cannot exceed the standard deviation,

(C) The root mean square deviation about a point A is least when A is the median,

(D) The mean deviation about a point A is minimum when A is the arithmetic mean.

30. Consider the following classical linear regression of y on x,

$$y_i = \beta x_i + u_i, i = 1, 2, \ldots, \ldots, n$$

where $E(u_i) = 0,\ V(u_i) = \sigma^2$ for all i, and $u_i's$ are homoscedastic and non-autocorrelated. Now, let $\hat{u}_i$ be the ordinary least square estimate of u_i. Then which of the following statements is true?

(A) $\sum_{i=1}^{n} \hat{u}_i = 0$,
(B) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i = 0$,
(C) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i \neq 0$,
(D) $\sum_{i=1}^{n} x_i \hat{u}_i = 0$.

Syllabus for PEB (Economics), 2013

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for PEB (Economics), 2013

1. An agent earns w units of wage while young, and earns nothing while old. The agent lives for two periods and consumes in both the periods. The utility function for the agent is given by $u = \log c_1 + \log c_2$, where c_i is the consumption in period $i = 1, 2$. The agent faces a constant rate of interest r (net interest rate) at which it can freely lend or borrow,

   (a) Find out the level of saving of the agent while young.

   (b) What would be the consequence of a rise in the interest rate, r, on the savings of the agent?

2. Consider a city that has a number of fast food stalls selling Masala Dosa (MD). All vendors have a marginal cost of Rs. 15/- per MD, and can sell at most 100 MD a day.

   (a) If the price of an MD is Rs. 20/-, how much does each vendor want to sell?

   (b) If demand for MD be $d(p) = 4400 - 120p$, where p denotes price per MD, and each vendor sells exactly 100 units of MD, then how many vendors selling MD are there in the market?

   (c) Suppose that the city authorities decide to restrict the number of vendors to 20. What would be the market price of MD in that case?

   (d) If the city authorities decide to issue permits to the vendors keeping the number unchanged at 20, what is the maximum that a vendor will be willing to pay for obtaining such a permit?

3. A firm is deciding whether to hire a worker for a day at a daily wage of Rs. 20/-. If hired, the worker can work for a maximum of 10 hours during the day. The worker can be used to produce two intermediate inputs, 1 and 2, which can then be combined to produce a final good. If

the worker produces only 1, then he can produce 10 units of input 1 in an hour. However, if the worker produces only 2, then he can produce 20 units of input 2 in an hour. Denoting the levels of production of the amount produced of the intermediate goods by k_1 and k_2 , the production function of the final good is given by $\sqrt{k_1 k_2}$. Let the final product be sold at the end of the day at a per unit price of Rs. 1/-. Solve for the firms optimal hiring, production and sale decision.

4. A monopolist has contracted with the government to sell as much of its output as it likes to the government at Rs. 100/- per unit. Its sales to the government are positive, and it also sells its output to buyers at Rs. 150/- per unit. What is the price elasticity of demand for the monopolists services in the private market?

5. Consider the following production function with usual notations.

$$Y = K^{\alpha} L^{1-\alpha} - \beta K + \theta L \text{ with } 0 < \alpha < 1, \beta > 0, \theta > 0.$$

Examine the validity of the following statements.

(a) Production function satisfies constant returns to scale.

(b) The demand function for labour is defined for all non-negative wage rates.

(c) The demand function for capital is undefined when price of capital service is zero.

6. Suppose that due to technological progress labour requirement per unit of output is halved in a Simple Keynesian model where output is proportional to the level of employment. What happens to the equilibrium level of output and the equilibrium level of employment in this case? Consider a modified Keynesian model where consumption expenditure is proportional to labour income and wage-rate is given. Does technological progress produce a different effect on the equilibrium level of output in this case?

7. A positive investment multiplier does not exist in an open economy simple Keynesian model when the entire amount of investment goods is supplied from import. Examine the validity of this statement.

8. A consumer consumes two goods, x_1 and x_2, with the following utility function

$$U(x_1, x_2) = U_1(x_1) + U_2(x_2).$$

Suppose that the income elasticity is positive. It is claimed that in the above set-up all goods are normal. Prove or disprove this claim.

9. A consumer derives his market demand, say x, for the product X as $x = 10 + \frac{m}{10p_x}$, where $m > 0$ is his money income and p_x is the price per unit of X. Suppose that initially he has money income $m = 120$, and the price of the product is $p_x = 3$. Further, the price of the product is now changed to $p'_x = 2$. Find the price effect. Then decompose price effect into substitution effect and income effect.

10. Consider an otherwise identical Solow model of economic growth where the entire income is consumed.

    (a) Analyse how wage and rental rate on capital would change over time.

    (b) Can the economy attain steady state equilibrium?